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Phil. Trans. R. Soc. Lond. A 1995 **353**, 115-126

doi: 10.1098/rsta.1995.0094

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Engineering chaos for encryption and broadband communication

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We present the different methods that have been proposed in the literature for sending information by means of a chaotic signal. They are based on three different ways to synchronize a receiver system with a chaotic transmitter system. Three different methods to modulate the transmitter system with an information carrying signal are also presented and some of their advantages and drawbacks are discussed.

1. Introduction

Chaotic behaviour has received much attention and created much enthusiasm in the scientific community for more than two decades, but the engineering community has been slow in accepting and studying this phenomenon. Furthermore, chaos has been looked upon by engineers as a mere disturbing factor that must be eliminated from any circuit or system that is intended to be of practical value. Only very recently, some engineers have begun to realize that the rich dynamics of chaos could be used in engineering. The most obvious application is the generation of pseudo-random signals. A more sophisticated use of chaotic behaviour has been proposed for communications, for control applications, for pattern recognition and for measuring devices. Communications on chaotic carrier signals is the subject of this paper. It is at present the most advanced application area for chaotic systems. An earlier presentation of similar material can be found in Hasler (1994a).

The general set-up for such a communication system is represented in figure 1. An information carrying signal $s(t)$ is injected into the transmitter system who shows chaotic behaviour. It generates an output signal of chaotic nature which is transmitted. The receiver is driven by this signal and performs the inverse operation of the transmitter, it retrieves the information signal. Thus the transmitter mixes in some way the information with chaos so that it is not possible, or very difficult, to extract the information from the transmitted signal. Another useful property of the transmitted signal is its broadband nature, as in the case of spread spectrum transmission.

It might be difficult to imagine how the receiver can extract from a chaotic signal the hidden information. Often, the receiver system is similar to the transmitter system and its behaviour is chaotic when it is not driven. To be able to retrieve the information, it is necessary to synchronize the two systems. We will discuss three methods for the synchronization of chaotic systems in § 2: (a) synchronization by decomposition into subsystems; (b) synchronization by linear feedback; (c) synchronization of the inverse system.

Phil. Trans. R. Soc. Lond. A (1995) **353**, 115–126

Printed in Great Britain

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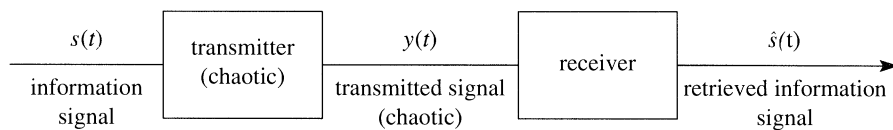


Figure 1. General set-up of the communication system.

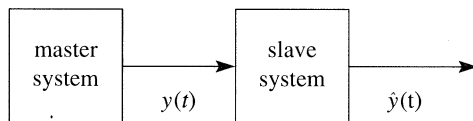


Figure 2. Master–slave set-up for synchronization.

In §3 we shall present three methods to hide/retrieve information in/from a chaotic carrier: (a) chaotic masking; (b) chaotic switching; (c) direct chaotic modulation.

The discussion will reveal that the design of such transmission systems is very delicate because of the conflicting design objectives robustness and security.

2. Synchronization of chaotic systems

The notion of synchronization is usually linked to periodic motion. Two periodic signals are synchronized, if their periods are identical. This definition clearly is of no use in the context of chaotic signals. In this case we require that the signals are identical, at least asymptotically when $t \rightarrow \infty$.

We shall consider a master-slave set-up as shown in figure 2. Mutual interaction for synchronization has also been studied in the literature, but in the context of communications it is less important.

Definition 1. The slave system *synchronizes* with the master system if

$$|\hat{y}(t) - y(t)| \rightarrow 0 \quad \text{as } t \rightarrow \infty \quad (1)$$

for any combination of initial states of the master and the slave system.

This definition can be extended to include approximate synchronization to accommodate inaccurate system parameters and non-ideal signal transmission. This topic is beyond the scope of the paper.

It might come as a surprise that it is possible to synchronize two systems with chaotic behaviour. Indeed, such systems have sensitive dependence on initial conditions, i.e. any two solutions drift apart, even if their initial conditions are very close to each other. It is the driving signal that forces the slave system to follow the time evolution of the master system. We now present the three methods to achieve this.

(a) Synchronization by decomposition into subsystems

The idea of synchronization by decomposition into subsystems has first been proposed in Pecora (1990). The following exposition of the method is inspired by Tesi (1993).

Suppose a nonlinear dynamical system is described by state equations of the form,

$$\frac{dx_1}{dt} = f_1(\mathbf{x}, \mathbf{y}_1), \dots, \quad \frac{dx_n}{dt} = f_n(\mathbf{x}, \mathbf{y}_1), \quad \frac{dy_1}{dt} = g_1(x_1, \mathbf{y}) \dots, \quad \frac{dy_m}{dt} = g_m(x_1, \mathbf{y}), \quad (2)$$

Encryption and broadband communication

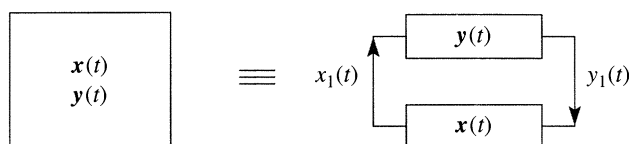


Figure 3. Decomposition of the system.

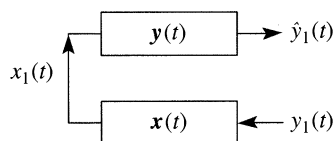
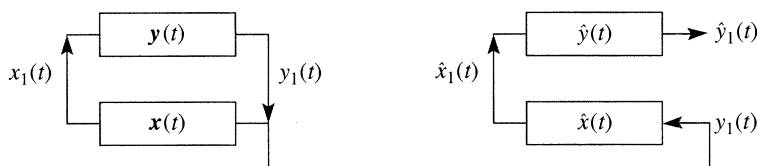
Figure 4. Decomposition cancelling the y -interaction.

Figure 5. Master-slave set-up for synchronization by decomposition into subsystems.

where $\mathbf{x} = (x_1, \dots, x_n)$ and $\mathbf{y} = (y_1, \dots, y_m)$. The system can be decomposed into two subsystems that interact only through the signals x_1 and y_1 (figure 3). To this decomposition corresponds the simple separation of the equations (2) into two groups, the first n equations that are the state equations for the variables x_i and the following m equations, the state equations for the variables y_i .

If we now cut the interaction by the signal y_1 , we obtain the two subsystems connected in cascade, as shown in figure 4.

This corresponds to a separation of the system of state equations (2) into two parts. Indeed, the lower system is described by

$$\frac{dx_1}{dt} = f_1(\mathbf{x}, y_1), \dots, \quad \frac{dx_n}{dt} = f_n(\mathbf{x}, y_1), \quad (3)$$

and the upper system is described by

$$\frac{d\hat{y}_1}{dt} = g_1(x_1, \hat{\mathbf{y}}) \dots, \quad \frac{d\hat{y}_m}{dt} = g_m(x_1, \hat{\mathbf{y}}), \quad (4)$$

where $\hat{\mathbf{y}} = (\hat{y}_1, y_2, \dots, y_m)$.

We now would like to synchronize the system of figure 4 with the system of figure 3. For this purpose, we transmit the signal $y_1(t)$, as represented in figure 5. Here, $\hat{\mathbf{x}} = (\hat{x}_1, \dots, \hat{x}_n)$ and $\hat{\mathbf{y}} = (\hat{y}_1, \dots, \hat{y}_m)$.

If both the systems in figure 5 started exactly at the same initial conditions $\mathbf{x}(0) = \hat{\mathbf{x}}(0)$, $\mathbf{y}(0) = \hat{\mathbf{y}}(0)$, then clearly the time evolution of the state variables in both systems would be identical, i.e. the two systems would be perfectly synchronized at all times. However, in most practical situations, we have no control over the initial conditions and therefore synchronization may or may not take place.

Let us now look at one of the numerous possible realizations of this synchronization scheme. As a system with chaotic behaviour, we take Chua's circuit (figure 6) described by the state equations

$$C_1 \frac{dv_1}{dt} = \frac{1}{R} (v_2 - v_1) - g(v_1), \quad (5)$$

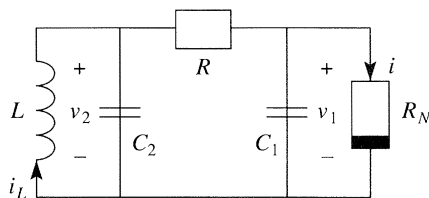


Figure 6. Chua's circuit.

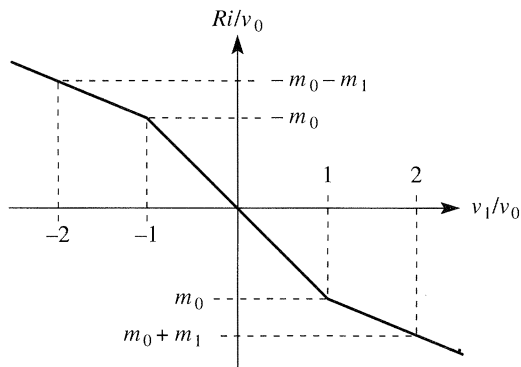


Figure 7. Characteristic of the nonlinear resistor in Chua's circuit.

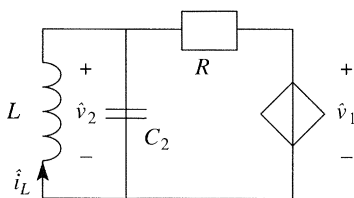


Figure 8. Subcircuit driven by the master circuit.

$$C_2 \frac{dv_2}{dt} = -\frac{1}{R}(v_2 - v_1) + i_L, \quad (6)$$

$$L \frac{di_L}{dt} = -v_2, \quad (7)$$

with the nonlinear resistor characteristic of figure 7. We choose the parameters to be $R = 1730 \Omega$, $L = 18 \text{ mH}$, $C_1 = 10 \text{ nF}$, $C_2 = 100 \text{ nF}$, $v_0 = 1 \text{ V}$, $m_0 = -0.44$, $m_1 = -0.23$, $r = 150 \text{ k}\Omega$.

The roles of the state variables \mathbf{x} and \mathbf{y} in figure 3 are played by $\mathbf{x} = (v_2, i_L)$, $\mathbf{y} = (v_1)$. The lower subsystem is shown in figure 8. It is driven by the complete circuit through transmission of the signal $y_1 = v_1$. It is linear and its elements have positive values. Thus it is globally asymptotically stable, which implies that its state variables \hat{i}_L and \hat{v}_2 converge to the state variables i_L and v_2 of the master circuit, as $t \rightarrow \infty$.

The upper subsystem of figure 3 is shown in figure 9. Suppose first that it is driven by the state variable $x_2 = v_2$ of the master circuit. Its state variable \hat{v}_1 may or may not converge to the state variable v_1 of the master circuit when $t \rightarrow \infty$, depending on the circuit parameters (Dedieu 1993). For the parameter set given above, numerical simulations indicate that synchronization takes place. For other

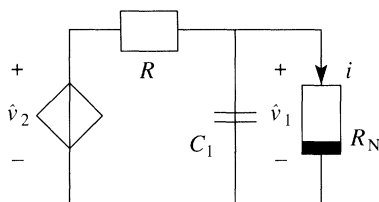


Figure 9. Subcircuit driven by the subcircuit of figure 8.

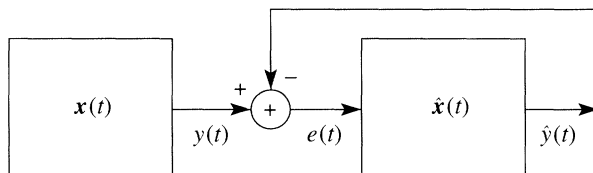


Figure 10. Master–slave set-up for synchronization by linear feedback.

parameter sets, where the circuit still exhibits chaotic behaviour, it is possible to prove synchronization rigorously, using a Lyapunov function (De Angeli 1994).

(b) Synchronization by linear feedback

This is the typical automatic control approach. We consider two identical systems as master and slave, compare their outputs and use the difference to control the slave system (figure 10). This approach has been introduced in (Chen 1993*a, b*) under the topic *control of chaos*.

If the output signal is a linear combination of the states and if the error signal controls the state variables linearly, the state equations of the whole system of figure 10 become

$$\frac{d\mathbf{x}}{dt} = f(\mathbf{x}), \quad (8)$$

$$y(t) = \mathbf{c}^T \mathbf{x}(t), \quad (9)$$

$$\frac{d\hat{\mathbf{x}}}{dt} = f(\hat{\mathbf{x}}) + \mathbf{k}e(t), \quad (10)$$

$$\hat{y}(t) = \mathbf{c}^T \hat{\mathbf{x}}(t), \quad (11)$$

$$e(t) = y(t) - \hat{y}(t). \quad (12)$$

Again, if both the master and the slave system started from exactly the same initial conditions then at all times, $\mathbf{x}(t) = \hat{\mathbf{x}}(t)$, $y(t) = \hat{y}(t)$, $e(t) \equiv 0$. With no constraint on the initial conditions, however, the slave system may or may not synchronize with the master system. In some instances, synchronization can be proved by Lyapunov functions (Chen 1993*a*; Hasler 1994*b*).

(c) Synchronization of the inverse system

In this method, the set-up of figure 1 is considered directly. The receiver is an *inverse system* of the transmitter system in the sense that for suitable initial conditions, the signal $\hat{s}(t)$ retrieved from the receiver is identical to the signal $s(t)$ injected into the transmitter. If we start from a different initial condition, we hope that

$$|\hat{s}(t) - s(t)| \xrightarrow[t \rightarrow \infty]{} 0. \quad (13)$$

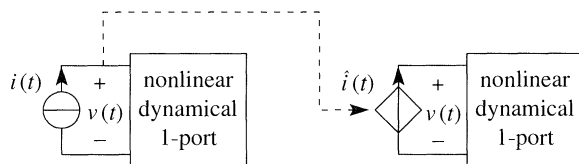


Figure 11. Realization of the inverse system by circuits.

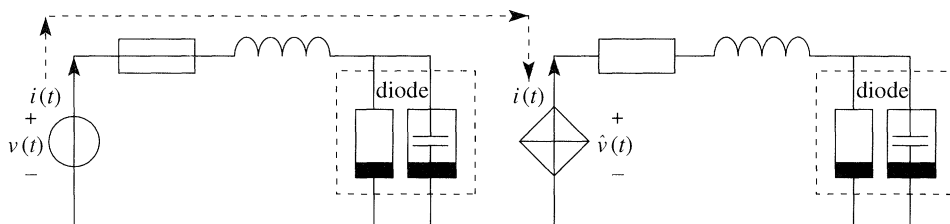


Figure 12. Synchronization of the RL-diode circuit by the inverse system.

If this is the case, we say that the *inverse system synchronizes* with the original system.

At first sight, it might seem a formidable task to find an inverse for a nonlinear dynamical system. In the context of circuits, however, there are evident candidates for inverse systems. Consider a nonlinear dynamical 1-port excited by an independent current source with current $i(t)$. If we take the voltage across the current source and inject it into another copy of the same 1-port by a voltage controlled voltage source, we usually will find exactly the same current $i(t)$ flowing through the voltage source, provided the initial currents in the inductors and the initial voltages across the capacitors in the two 1-ports are the same (figure 11). Dually, the signal $s(t)$ can be applied to the 1-port by means of a voltage source. In this case the current across the voltage source is transmitted and the voltage across the controlled current source in the slave circuit is retrieved.

Non-autonomous chaotic circuits can be directly used for the master system in figure 11. In (Boehme 1994) the dual system of figure 11 is implemented with the RL-diode circuit (figure 12). In this case, synchronization can be proved, because in the inverse system the current is imposed on the diode. Assuming that the nonlinear resistor and capacitor characteristics in the equivalent circuit of the diode are strictly increasing, it is not difficult to prove that $(q_1(t) - q_2(t))^2$ is strictly decreasing as a function of time, where $q_1(t)$ and $q_2(t)$ are the capacitor charges of two arbitrary circuit solutions, and that this implies unique asymptotic behaviour for the inverse system.

In the case of autonomous circuits, an independent voltage or current source is added in such a way that the chaotic behaviour is not destroyed. Synchronization of the inverse system has been achieved with Chua's circuit and with Saito's circuit (Halle 1993; Hasler 1993). In Halle, it was possible to prove synchronization rigorously, because the nonlinear resistor and the linear part of the slave circuit have their voltage imposed by the controlled source (figure 13). Therefore, $\hat{i}_1(t) = i_1(t)$ at all times and

$$|\hat{i}_2(t) - i_2(t)| \xrightarrow[t \rightarrow \infty]{} 0.$$

since i_2 is the port current of a linear passive 1-port. In other words, the inverse system is globally asymptotically stable. This implies that $\hat{i}(t)$ synchronizes with $i(t)$.

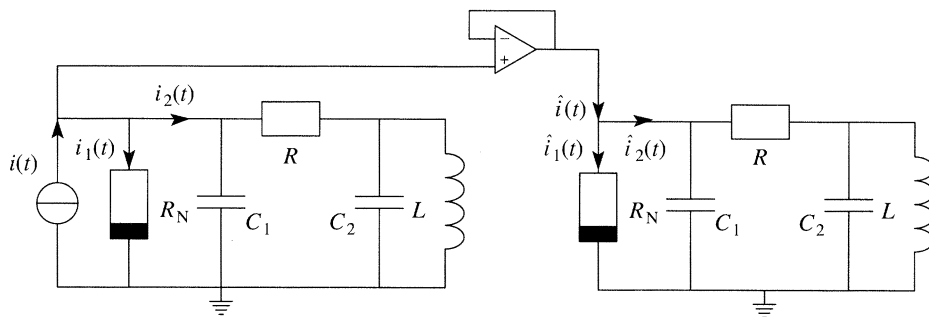


Figure 13. Synchronization of Chua's circuit by the inverse system.

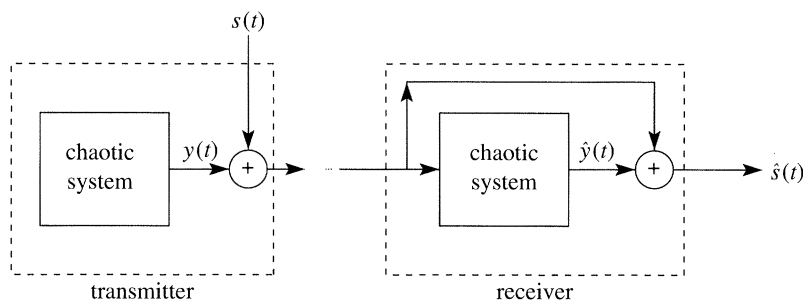


Figure 14. Transmission using chaotic masking.

In the set-up of Hasler (1993), neither the voltage nor the current of the nonlinear element is imposed, and therefore the proof of synchronization appears to be difficult. Nevertheless, we have confirmed synchronization by computer simulation.

3. Transmission of information by a chaotic signal

(a) Chaotic masking

In this method (Oppenheim 1992; Kocarev 1993) an analogue information carrying signal $s(t)$ is added to the output $y(t)$ of the chaotic system in the transmitter. On the receiver side an identical chaotic system tries to synchronize with $y(t)$. From this point of view, the information signal $s(t)$ is a perturbation and synchronization will take place only approximately. However, if the synchronization error is small with respect to $s(t)$, the latter can be approximately retrieved by subtraction (figure 14). This is the case if the signal $s(t)$ is small with respect to $y(t)$ and/or if the spectra of the two signals do not overlap too much. Both of these requirements can apparently be relaxed (Lozi 1993; Cuomo 1993). However, if the purpose of using a chaotic signal for transmission is to hide the information, $s(t)$ should not be large. Therefore, it can be expected that the method is sensitive to channel noise. Indeed, additive noise cannot be distinguished from $s(t)$ by the set-up of figure 14 and it has to be eliminated at a later stage. This is a difficult if not impossible task if the amplitude of $s(t)$ is not large with respect to the noise level.

(b) Chaotic switching

In this method the information signal $s(t)$ is supposed to be binary. It controls a switch whose action changes the parameter values of the chaotic system. Thus,

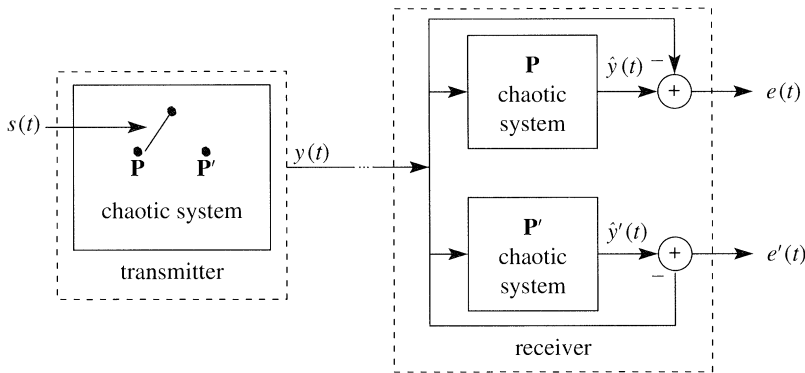


Figure 15. Transmission using chaotic switching.

according to the value of $s(t)$ at any given instant t , the chaotic system has either the parameter vector \mathbf{p} or the parameter vector \mathbf{p}' . The output $y(t)$ of the chaotic system is transmitted to two copies of the chaotic system, one with the parameter vector \mathbf{p} and the other with the parameter vector \mathbf{p}' (figure 15).

If the momentary position of the switch in the transmitter is on position \mathbf{p} , then the system with parameter vector \mathbf{p} in the receiver will synchronize, whereas the system with parameter vector \mathbf{p}' will desynchronize. Thus the error signal $e(t)$ will converge to zero, whereas $e'(t)$ will have an irregular wave form with a distinctly non-zero amplitude. If the switch in the transmitter is on position \mathbf{p}' , then we have the opposite situation, $e'(t)$ will converge to zero and $e(t)$ will be of non-zero amplitude. Consequently, the signal $s(t)$ can be retrieved from the error signals $e(t)$ and $e'(t)$. Clearly, one has to leave the switch in the transmitter a certain time in the same position to be able to observe the convergence of the corresponding error signal to zero.

In some realizations only one chaotic system is used on the receiver side. To distinguish the transmitted bit value, one has to decide between synchronization and desynchronization on the basis of a single error signal.

Chaotic switching has been realized with Chua's circuit, switching a linear resistor in parallel with the nonlinear resistor and performing the synchronization with decomposition into subcircuits (Dedieu 1993). In figure 16, the transmitted signal, the voltage v_1 (cf. figure 6), is plotted against the retrieved voltage \hat{v}_1 of both systems in the receiver. Clearly, one of them synchronizes since the value of v_1 and \hat{v}_1 is nearly identical at all times and thus the $v_1 - \hat{v}_1$ plot remains close to the diagonal. This is not at all the case for the other system in the receiver, which renders its desynchronization visible. These curves have been gathered from an experimental set-up. In figure 17, the simulated time evolution of the transmitted signal, the voltage of capacitor is represented when the resistor is periodically switched on and off, as indicated by the dashed line. The irregular nature of the capacitor voltage does not allow to retrieve this binary information in any evident way.

In Hasler (1994b), the same parameter is switched, but synchronization is performed by linear error feedback, whereas in Parlitz (1993), the capacitors and the inductor are switched between two values and synchronization is achieved by decomposition into subcircuits. The method of chaotic switching has also been described in Bel'skii (1993), with a different chaotic system, synchronization by decomposition into subsystems and a single chaotic circuit on the receiver side.

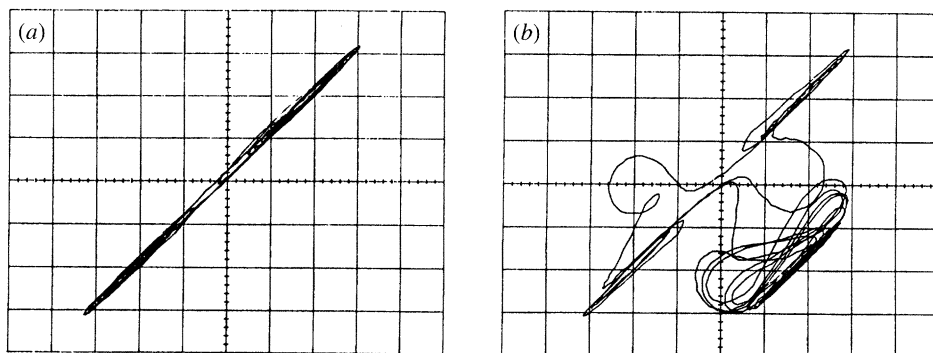


Figure 16. Transmitted against retrieved voltage v_1 for the receiver subsystem with the right parameters (left) and with the wrong parameters (right) (from Dedieu 1993).

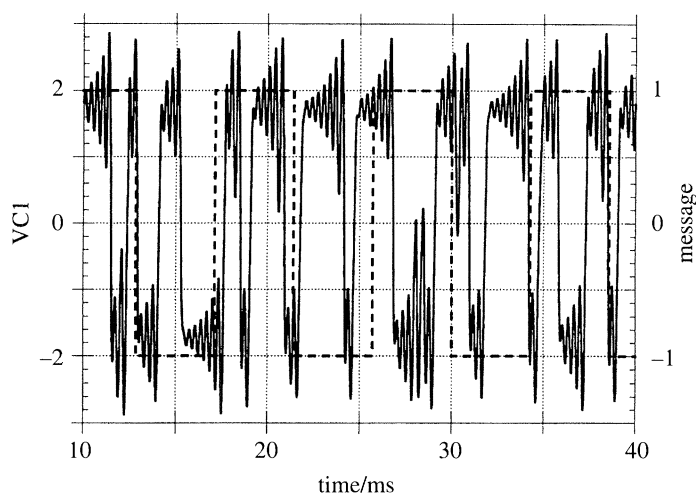


Figure 17. Transmitted signal and hidden binary signal (from Dedieu 1993).

While chaotic switching is expected to be more robust against noise than chaotic masking, its information transmission rate is lower, because on the one hand, the binary signal has a lower information content per unit of time than the analogue signal and on the other hand, for each bit that is transmitted, one has to wait until synchronization and desynchronization is achieved in the receiver.

(c) Direct chaotic modulation

This method uses the general set-up of figure 1 in a straightforward way. The signal $s(t)$ is the information carrying signal and $y(t)$ is the transmitted signal. Thus, no additional circuitry has to be used, the chaotic system is the transmitter and the inverse system is the receiver. If we look at a circuit realization, e.g. in figure 12 we can see that $s(t)$ drives the chaotic circuit and thus modulates the chaotic signal in some way.

The information can be injected directly in analogue form, as proposed in Halle (1993), or $s(t)$ can be itself an analogue signal modulated by binary information, as proposed in Hasler (1993), with the obvious advantages and drawbacks. The trans-

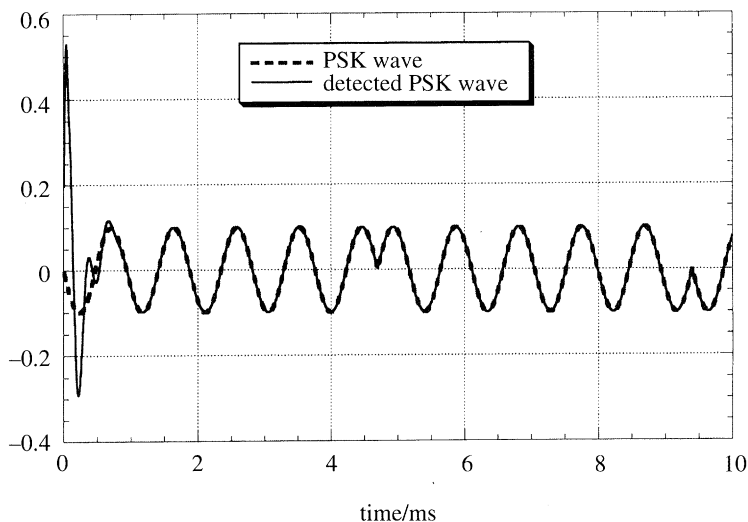


Figure 18. Original and retrieved information signal, for direct modulation with Saito's circuit (from Hasler 1993).

mission of a digital signal modulated on to $s(t)$ can be expected to reach higher bit-rates than with chaotic switching. In chaotic switching, whenever the signal changes its value, one has to wait for synchronization since the initial conditions in the transmitter and the receiver subsystem that has to synchronize are different. In direct chaotic modulation, the receiver continuously tracks the transmitter and thus the states of the two chaotic systems are never very different. This can be seen in figure 18, where a phase modulated signal is transmitted on a chaotic carrier, using Saito's circuit. At the beginning, the receiver needs some time to synchronize, but afterwards the receiver tracks the 180° phase shifts perfectly. In figure 19, the transmitted signal is represented for the same experiment. Both figures have been obtained by computer simulation.

4. Conclusions

Various methods have been presented which permit to transmit information through a chaotic signal. For this purpose it is necessary to have a transmitter with chaotic behaviour who produces such a signal and a receiver with unique asymptotic behaviour who is able to synchronize with the transmitter. Furthermore, the information carrying signal has to modulate the chaotic carrier signal in some way. Three methods for synchronization and three methods for modulation have been presented. If we look at them from the point of view of cryptography, the secret key is the value of the circuit parameters. Therefore, the systems should be designed in such a way, that synchronization does not occur anymore if the parameters are not correct. On the other hand, synchronization should still take place, approximately, if the parameters are slightly inaccurate, so that a real system can work. This leads to a delicate design problem, a difficult compromise between security and robustness.

Further developments are needed if the systems proposed in this paper are to compete with conventional cryptographic or spread spectrum systems. In particular, it should be investigated to which extent the circuit parameters of the transmitter

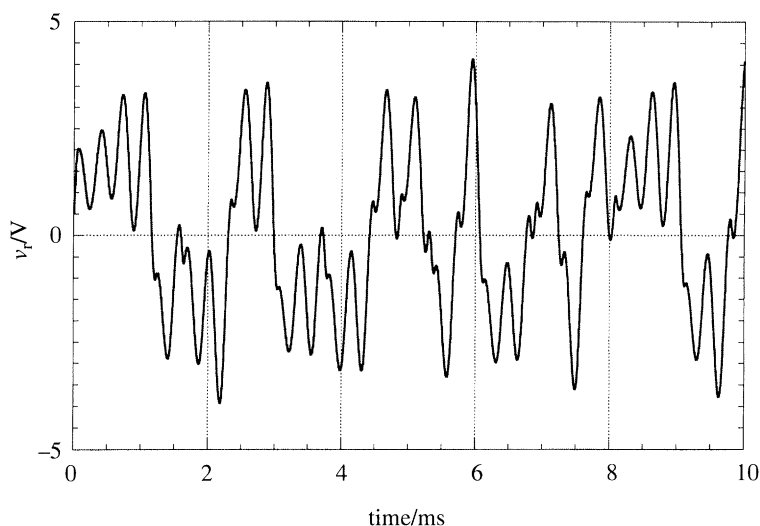


Figure 19. Transmitted signal, for the case of direct modulation by Saito's circuit.

can be identified, what influence the channel noise and the channel parameters have, and how a source separation can be performed when more than one transmitter is sending on the same medium, as in CDMA systems.

Notwithstanding these problems, it is fascinating to see how the rich dynamics of chaotic systems may find their way into engineering applications.

The author is indebted to H. Dedieu for many stimulating discussions and to the Swiss National Science Foundation for financial support (grant no. 2000-040705.94/1).

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